Objectives:

- Review the chain rule for derivatives.
- Define and use u-substitution for integrals.

Review: Compute the following derivatives:

1. $h(x) = f(g(x))$

   $h'(x) =$

2. $f(x) = (x^3 + 2x)^5$

   $f'(x) =$

3. $g(x) = e^{x^2}$

   $g'(x) =$

4. $h(t) = (\sin(t))^3$

   $h'(t) =$

5. $k(x) = \ln(\tan(x))$

   $k'(x) =$

6. $\ell(w) = \arctan(\cos(w))$

   $\ell'(w) =$

Substitution:

So, we now know $\int \frac{(\sec(x))^2}{\tan(x)} dx =$ ____________________________ .

In general, $\int f'(g(x))g'(x)\, dx =$ ____________________________ .

Another way we can write this is $\int f'(u)\, du =$ ______________ . We call this substituting u for $g(x)$ and often refer to this method as _____________________ .

Examples:

1. $\int e^{\sin(x)} \cos(x)\, dx$

2. $\int 3x^2 \left(x^3+5\right)^{10}\, dx$

3. $\int \frac{1}{t+2} dt$

Sometimes we have to manipulate the integral before using substitution:

Example: $\int x^2 \left(x^3 + 7\right)^5 \ dx$

Method 1: Solve for dx:

Method 2: "Fix-it-up":

Definite Integrals Using u-Substitution:

Example: $\int_0^1 x^2 \left(1 + 2x^3\right)^4 dx$

Method 1 : Change limits of integration:

Method 2 : Find integral in terms of original variable, then substitute.